Многокритериална оптимизация за производството на етанол от щам *Saccharomyces cerevisiae*

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Multiple Objective Optimisation for the Ethanol Production from Strain *Saccharomyces sereviciae*

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Abstract. *A fuzzy procedure is applied in order to find the optimal feed policy of a fed-batch fermentation process for ethanol production using a Saccharomyces serevicieae. The policy consists in feed flow rate, feed concentration, and fermentation time. In this study biotechnological process is formulated as a general multiple objective optimisation problem. By using an assigned membership function for each of the objectives, the general multiple objective optimisation problem can be converted into a maximizing decision problem. In order to obtain a global solution, a method of fuzzy sets theory is introduced to solve the maximizing decision problem. After this multiple optimisation, the useful product quality is raised and the residual substrate concentration is decreased at the end of the process. Thus, the process productiveness is increased.*

1. Introduction

The ethanol is the most important organic compound which has a wide application in different industry fields: food, perfumery-cosmetic, chemical, millwright, etc. In recent 10–15 years enormous attention is paid to the ethanol production as a fuel. Ethanol production from renewable resources can improve energy security, reduce accumulation of carbon dioxide, and decrease urban air pollution. When blended with gasoline, “neat” ethanol reduces the release of smogforming compounds. Thus, ethanol from lignocellulosic materials holds great promise as a new industry in the world and has the potential for making a significant contribution to the solution of major energy as well as environmental problems. Although the ethanol production using a fermentation of sugar has been studied for many years, there are several bottlenecks for the economical production of fuel ethanol. One of them is ethanol inhibition, which is considered to be the principal factor restricting the fermentation rate and concentration of ethanol achievable in the production process. Lignocellulosic feedstocks like wood, waste paper, and fast-growing energy crops have been identified as economical starting materials for ethanol production [1-3].

Multiple objective optimisation is a natural extension of the traditional optimisation of a single objective function. If the multiple objective functions are commensurate, minimizing single objective function, it is possible to minimize all criteria and the problem can be solved using traditional optimisation techniques. On the other hand, if the objective functions are incommensurate, or competing, then the minimization of one objective function requires a compromise in another objective function. The competition between multiple objective functions is a key distinction between multiple objective optimisation and traditional single-objective optimisation [4,5].

Vera at all [6] have illustrated a general multiple objective optimisation framework of biochemical systems and have applied it optimizing of several metabolic responses involved in the ethanol production process by using *Saccharomyces cerevisiae* strain.

Tonnon at all [7] have used interactive procedure to solve multiple-objective optimization problems. A fuzzy set has been used to model the engineer’s judgment on each objective function. The properties of the obtained compromise solution were investigated along with the links between the present method and those based fuzzy logic. An uncertainty which has been affecting the parameters is modelled by means of fuzzy relations or fuzzy numbers, whose probabilistic meaning is clarified by random set and possibility theory. Constraint probability bounds that satisfy a solution can be calculated and procedures that consider the lower bound as a constraint or as an objective criterion are presented. Some theorems make the computational effort particularly limited on a vast class of practical problems. The relations with a recent formulation in the context of convex modelling are also pressured.

Wang at all [8] have used a fuzzy decision making procedure to find the optimal feed policy of a fed-batch fermentation process for fuel ethanol production, by using an assigned membership function for each of the objectives, the general multiple objective optimisation problem is converted into a maximizing decision problem.

In this paper we apply the ideas of multiple objective optimisation to the design of an optimal feed policy for a fed-batch fermentation process for an ethanol production from glucose using the *Saccharomyces cereviziae* strain.

2. Material and Methods

2.1. Processes Specific

The following assumptions are made in developing of the model of the aerobic baker's yeast growth process in batch and fed-batch cultures [9]:

- The main by-products in an aerobic yeast growth process are water, carbon dioxide and ethanol.
- The bioreactor is a perfectly mixed.
- Ethanol consumption is inhibited when sugar concentration in the broth is higher than a critical level.
- The elemental composition of yeast in the process does not significantly change.
- Parameters except for the substrate and product concentrations, e.g. pH and temperature, are controlled to certain acceptable constant values during the process.

The cultivation parameters are:

➢ Temperature	30°C;
➢ pH	5.4;
➢ Gassing rate	275 L/h air;
➢ Stirrer speed at start	800 rpm;
➢ Volume	1.5 L;
➢ Glucose	0.5 g/L;
➢ Total time	T = 16 h.

Experimental data from three batch cultivations of baker's yeast, obtained in the Institute of Technical Chemistry, University of Hannover, are used. The experimental data consist off-line measurements of biomass (yeast), substrate (glucose) and ethanol. The bacterial growth in batch cultivation was monitored at measuring optical density of 1-ml samples spectrophotometrically at 600 nm.

2.2. Kinetic Model

The kinetic model of the process is [10]:

$$(1)\quad \begin{aligned} \frac{dX}{dt} &= \mu_m \frac{S}{k_s+S} X - \frac{F}{V} X \\ \frac{dS}{dt} &= \frac{F}{V}(S_{in}-S) - \frac{1}{Y_S}\frac{\mu_m S}{k_s+S} X \\ \frac{dE}{dt} &= \frac{1}{Y_E}\frac{\mu_m S}{k_s+S} X - \frac{F}{V} E \\ \frac{dV}{dt} &= F \end{aligned}$$

where: X – cell concentration, g/l; S – substrate concentration, g/l; E – ethanol concentration, g/l; S_{in} – feed concentration, g/l; t – time, h; $Y_{S\,E}$ – yield coefficients of the substrate and the ethanol; μ_m – maximum specific growth rate of biomass, h^{-1}; k_s – the Michaelis-Menten constant; V – working volume of the bioreactor, l; F – feed flow rate, l/h.

The initial conditions and the kinetics constants is [10]:
$X(0) = X_0 = 0.27$ g/l; $S(0) = S_0 = 31.30$ g/l; $S_{in} = 100$ g/l;
$E(0) = E_0 = 0.47$ g/l; $V(0) = V_0 = 1.2$ l; $\mu_m = 0.342$ h^{-1};
$k_s = 5.436$ g/g; $Y_S = 5.762$ g/g; $Y_E = 1.985$.

2.2. System Constraints

Nearly all engineering processes will have physical constraints [12,13]. In this study, the flow rate is bounded and the volume of the bioreactor is constrained, i.e.

(2) $F_{min} \le F(t) \le F_{max}$;

(3) $g_1 = V(t) - V_F \le 0$.

The concentration of glucose must by positive all the time; otherwise, an unrealistic solution in the optimisation problem would be obtained. Therefore we have

(4) $g2 = -S(t) \le 0$.

In addition, the stoichiometry of the biomass formation from substrate (glucose), posing one constraint given must be as follows:

$$(5)\quad g_3 = \frac{X(t)\,V(t) - X_0\,V_0}{[V(t)-V_0]\,S_{in} + S_0\,V_0 - S(t)\,V(t)} - \frac{1}{Y_S} \le 0 \,.$$

If the constraints in (3)–(5) are not included in the optimisation problem, unrealistic predicted values may be found.

3. Multiple-objective Optimisation Problem

3.1. Formulation of the Multiple-objective

The objective of the problem is to find optimal feed flow rate – $F(t)$, feed concentration – S_{in}, and fermentation time – t_f of the fed-batch process, such that the ethanol production should be greater than or equal to a threshold value, the consumption of glucose should be less than or equal to a threshold value, and the fermentation time should also be less than or equal to a threshold value.

The multiple-objective optimisation problem (MOOP) is expressed this way:

$$(6)\quad \max_{F(t),S_m,t_f} J_1 = E(t_f)\,V(t_f) - E_0\,V_0\,;$$

$$(7)\quad \min_{F(t),S_m,t_f} J_2 = S_{in}\,(V(t_f) - V_0) + S_0\,V_0\,;$$

$$(8)\quad \min_{F(t),S_m,t_f} J_3 = t_f\,.$$

The first objective function corresponds to the total price of ethanol production. The second objective function is the cost of the substrate. The last objective function corresponds to the operation cost.

The concept of Pareto optimality or non inferiority is therefore used to characterize a solution to the multiple-objective optimisation problem. In order to concisely define the Pareto optimal solution, the following definitions are introduced [12-14].

Definition 1. The feasible region in input space, Ω is the set of all admissible control variables and the system parameters that satisfy the system constraints

$$\Omega = \{\mathbf{u} \equiv [F(t), S_{in}, t_f]^T \mid \dot{\mathbf{z}} = \mathbf{f}(\mathbf{z},\mathbf{u}),\ \mathbf{z}(0) = \mathbf{z}_0, \\ F_{\min} \le F(t) \le F_{\max};\ g_k(\mathbf{z},\mathbf{u}) \le 0,\ k = 1,...,3\}.$$

Here the state equation, $\dot{\mathbf{z}}=\mathbf{f}(\mathbf{z},\mathbf{u})$ consists of the fed-batch model in (1). The decision variable **u** consists in the $\mathbf{u}=\mathbf{u}[F(t), S_{in}, t_f]$.

We are now in a position to define Pareto optimal solutions to the combined optimal control and optimal parameter selection problem.

Definition 2. A control action **u*** is a Pareto optimal policy if and only if $\mathbf{u} \in \Omega$ such that [8] does not exist there:

$$J_i(\mathbf{u}) \le J_i(\mathbf{u}^*) \quad i = 1,...,3; \quad J_k(\mathbf{u}) < J_k(\mathbf{u}^*) \ \textit{for some } k.$$

In general, there is an infinite number of Pareto policies for a given multiple objective optimisation problem. The collection of Pareto policies is Pareto set. The image of this set is called the trade-off surface. The literature on multiple-objective optimisation is huge, and we cannot hope to mention all the techniques that have been used to generate Pareto solutions.

After the optimal solution is obtained through a multiple-objective optimisation technique, the second requirement in this decision-making problem (DM) is then performed to check whether or not the optimal solution satisfies the assigned threshold values. If the optimal solution does not satisfy the threshold values, DM has to assign another threshold requirement. The problem should then be repeated to find another optimal solution. Interactive programming can be employed to solve the decision-making problem. In this study, the interactive fuzzy optimisation is extended to solve the multiple-objective optimal control and optimal parameter selection problem.

3.2. General Multiple-objective Optimisation Problem

The multiple-objective optimisation problem (6)-(8) is now extended to the general multiple-objective optimisation problem (GMOOP) given as

$$(9)\quad \textit{fuzzy}\ \max_{\mathbf{u}} J_1 = E(t_f)\,V(t_f) - E_0 V_0;$$

$$(10)\quad \textit{fuzzy}\ \min_{\mathbf{u}} J_2 = S_{in}(V(t_f) - V_0) + S_0 V_0;$$

$$(11)\quad \textit{fuzzy}\ \min_{\mathbf{u}} J_3 = t_f.$$

The membership function of (9) has the type:

$$(12)\quad \eta_1(J_1) = \begin{cases} 0; & J_1 < J_1^L \\ \dfrac{J_1 - J_1^L}{J_1^U - J_1^L}; & J_1^L \le J_1 \le J_1^U \\ 1; & J_1 > J_1^U \end{cases}$$

where J_1^L or J_1^U represents the value of J_1 such that the grade of the membership function $\eta(J_1)$ is 0 or 1.

The membership functions for minimizing goals of (10)-(11) are expressed as

$$(13)\quad \eta_k(J_k) = \begin{cases} 1; & J_k \le J_k^L \quad k = 2,3 \\ \dfrac{J_k - J_k^L}{J_k^U - J_k^L}; & J_k^L < J_k < J_k^U \\ 0; & J_k \ge J_k^U \end{cases}$$

where J_k^L or J_k^U represents the value of J_k such that the grade of the membership function $\eta(J_k)$ is 1 or 0.

As a result, the DM considers the fuzzy objective function such as J_1 should be substantially greater than or equal to a threshold interval $[J_1^L\ J_1^U]$. The second and third, goals should be substantially less than or equal to the assigned threshold interval $[J_k^L\ J_k^U]$, k=2, 3.

The membership function for each of the objective functions is described on *figure 1*.

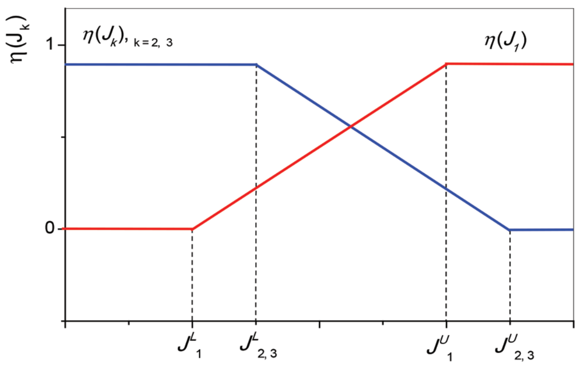


Figure 1. Assigned membership function for each of the objective functions

Having elicited the membership functions for each of the objective functions, the GMOOP (9)-(11) can be converted into the fuzzy multiple-objective optimisation problem (FMOOP) by [8]:

$$\min_{\mathbf{u}\in\Omega} \left[\eta_1(J_1)\ \ \eta_2(J_2)\ \ \eta_3(J_3)\right]^T.$$

By introducing a general aggregation function $\eta_D(J_k)$, a fuzzy multiple-objective decision making problem (FMODMP) or maximizing decision problem can be defined using this [8]:

$$(14)\quad \eta_D = \max_{\mathbf{u}\in\Omega} \min_k \{\, \eta_k(J_k),\ k = 1,...,3 \,\}.$$

Observe that the value of the aggregation function can be interpreted as representing an overall degree of satisfaction with the DM's multiple fuzzy goals. Let us consider the fuzzy maximizing problem. While the objective function value is greater than the assigned upper bound, such a solution absolutely satisfies the DM. On the other hand, the objective function value is less than the lower bound. It must be rejected. While the objective function value is located between the threshold interval, the DM has a degree of satisfaction for the solution.

Fundamental to the MOOP in (6)-(8) is the Pareto optimal concept, and thus the DM must select a compromise solution from many Pareto optimal solutions. The relationships between the optimal solutions of the FMODMP and the Pareto optimal concept of the MOOP can be characterized by the following theorem [8]:

Theorem 1. If **u*** is a unique optimal solution to the FMODMP in (14), then **u*** is a Pareto optimal solution to the MOOP in (6)-(8).

The proof of this theorem follows immediately from Definition 2 of the Pareto optimality by making use of contradiction arguments. This theorem is used to guarantee that the unique optimal solution of the FMODMP is a Pareto solution to the crisp multiple objective optimal control problems (6)-(8). The statement of this theorem does not guarantee that the unique optimal solution to (14) is a Pareto solution to the GMOOP (9)-(11).

Definition 3. If $\mathbf{u}^* \in \Omega$ is said to be a M-Pareto optimal solution to GMOOP if and only if another $\mathbf{u} \in \Omega$ does not exist there, such that $\eta_k(J_k(\mathbf{u}))\geq\eta_k(J_k(\mathbf{u}^*))$ for all k and $\eta_j(J_j(\mathbf{u}))\neq\eta_j(J_j(\mathbf{u}^*))$ for at least one j.

Note that the set of Pareto optimal solutions is a subset of the set of M-Pareto optimal solutions as observed from Definitions 2 and 3 and (12)-(13). Here M refers to membership. Using the concept of M-Pareto optimality, the fuzzy version of *Theorem 1* can be obtained under slightly different conditions.

Theorem 2. If **u*** is a unique optimal solution to the FMODMP (14), then **u*** is an M-Pareto optimal solution to the GMOOP (9)-(11).

The proof of this theorem follows immediately from *Definition 3* of the M-Pareto optimality by making use of contradiction arguments. *Theorem 2* is used to guarantee that the unique optimal solution of the maximizing decision problem (14) is a M-Pareto optimal solution of the fuzzy problems (9)-(11). The key point for using this theorem is to find a unique optimal solution of the problem (14). A global optimisation method must be employed to determine such a unique solution.

Interactive programming techniques are tools for searching a satisfactory solution by interaction between the DM and the computer. It can be regarded as an interface between humans and computers. An interactive programming algorithm is introduced in this study and is listed in the following to find a satisfactory solution to the GMOOP.

Algorithm

1. Assign the threshold intervals $[J^L_k\ J^U_k]^r$.
2. Elicit a membership function $\eta(J_k)$ from the DM for each of the objective functions.
3. Solve the maximizing decision problem (14). Let $(\mathbf{u}^r,\ \eta^r_k(J_k))$ be the M-Pareto optimal solution to the GMOOP.
4. If the DM is satisfied with the current levels of $\eta^r_k(J_k)$, k=1,...,3, the current M-Pareto optimal solution $(\mathbf{u}^r,\ \eta^r_k(J_k))$ is the satisfactory solution for the DM. Otherwise, classify the objectives into three groups based on the DM's preference, including (a) a class of the objectives that the DM wants to improve, (b) a class of the objectives that the DM may possibly agree to relax, and (c) a class of the objectives that the DM accepts.

The index set of each class is represented by $\mathbf{I}^r$, $\mathbf{R}^r$, and $\mathbf{A}^r$, respectively. The new threshold intervals $[J^L_k\ J^U_k]^{r+1}$ are reassigned in such a way that $[J^L_k\ J^U_k]^r\subset [J^L_k\ J^U_k]^{r+1}$ for any $k\in\mathbf{I}^r$, $[J^L_k\ J^U_k]^{r+1}\subset[J^L_k\ J^U_k]^r$ for any $k\in\mathbf{R}^r$, and $[J^L_k\ J^U_k]^{r+1}= [J^L_k\ J^U_k]^r$ for any $k\in\mathbf{A}^r$. Then repeat step 2.

Here, it should be stressed that any improvement for one of the objective functions can be achieved only at the expense of at least one of the other objective functions.

5. Results and Discussion

Since the feed rate F(t) is time dependent variable, the optimal control problem can be considered for an infinite dimensional problem. To solve this problem efficiently, the feed flow rate is represented by a finite set of control parameters in the time interval $t_{j-1} < t < t_j$ as follows F(t) = F(j) for j=1,...,K – number of time partitions.

The control variables are satisfied in the following intervals: $0 \leq F(t) \leq 20.0\times10^{-3}$ l/h; $80 \leq S_{in} \leq 150$ g/l and $10 \leq t_f \leq 16$ h.

Since the physical constraints in (3)-(5) are included in the optimisation problem, the penalty function method is used to handle the system constraints in *fuzzy optimisation* (FO). The function used in FO is, therefore, defined as

$$\max_{\mathbf{u}} J = \eta_D - \sum_{i=1}^{3}\lambda_i\int_0^{t_f}\langle g_i\rangle_+^2\,dt\ . \tag{15}$$

The integration of the square penalty functions in (15) is used to cover the state variables on the whole time domain.

The following optimisation problem in the class of the fuzzy mathematical programming problems can be formulated:

$$J \cong \widetilde{\max_{\mathbf{u}}} J\ , \tag{16}$$

where: $\widetilde{\max}$ means *in possibility maximum*; $\cong$ means *is come into view approximately in following relation*.

For determination of this problem, an approach generalizing the Bellman-Zadeh's method [15,16] is used:

$$\mu_D(J) = (1-\gamma)\,\eta(J)^\theta + \gamma[1-\left(1-\eta(J)\right)^\theta] \tag{17}$$

where: λ – parameter characterized the compensation degree; θ – parameter, those give weight of $\eta(J)$.

The solution is received using the common *defuzzification* method BADD [17]:

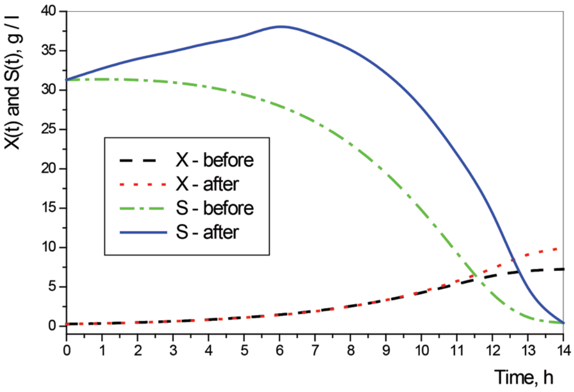
$$\mathbf{u}^0 = \sum_{i=1}^{q}\frac{\mu^\theta_{D_i}(J)\,\mathbf{u}_i}{\sum_{j=1}^{p}\mu^{\theta_i}_{D_j}(J)} \tag{18}$$

where: $i = 1, \ldots, q$; $j = 1, \ldots, p$, $p = q^m$; q – number of discrete values of the vector **u**.

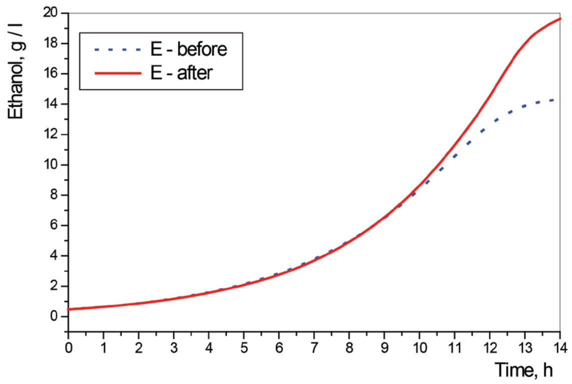
Now, the maximizing decision problem can be solved by fuzzy sets theory. The fuzzy algorithm was written by a COMPAQ Visual FORTRAN 90. All computations have been performed on an on an DualCore AMD Athlon II X2 245 2900 MHz computer using Microsoft Windows XP Pro Edition operating system.

The obtained results after optimisation are: S_{in} = 139 g/l and t_f = 14 h.

The results for the biomass, substrate and ethanol concentrations, are shown in *figure 2*. The feed flow rate is shown in *figure 3*.



2a) Biomass and substrate concentration



2b) Ethanol concentration

Figure 2. Biomass, substrate and ethanol concentration before and after optimisation

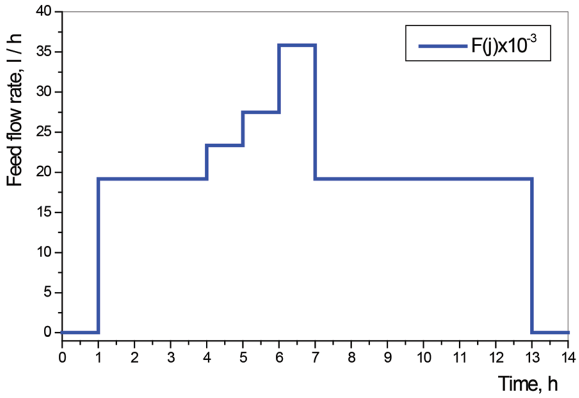


Figure 3. Optimal profile of feed flow rate

The obtained results (*figure 2a*) show increase of the biomass concentration with more than 36% (7.24 g/l – before optimisation, 9.92 g/l after optimisation. The residual substrate concentration decreases with more than 1.5% (0.428 g/l – before optimisation, 0.420 g/l after optimisation). *Figure 2b* shows an increase of the ethanol concentration with more than 37% (14.32 g/l – before optimisation and 19.623 g/l after optimisation).

Conclusions

A fuel ethanol production planning problem using a Saccharomyces serevicieae is discussed in this study. Such a production planning problem is formulated into a framework of the general multiple-objective optimisation problem. Many of the multiple-objective optimisation problems in the real world take place in an environment in which the goals, the constraints, and the consequences of possible actions are not known precisely. To deal with imprecision quantitatively, the problem in a fuzzy environment is introduced in this study to handle these imprecise goals and constraints. Such fuzzy multiple-objective optimal control problems are converted into a maximizing decision problem through the subjective membership functions for each of the objective functions. The optimal solution for each of the membership functions is denoted as the degree of satisfaction with the assigned threshold requirements.

In order to obtain a global optimal solution, a fuzzy sets theory method is introduced to solve the maximizing decision problem. A simple guideline is presented in the interactive programming procedures in order to find a satisfactory solution to the general multiple-objective optimisation problem.

The obtained results from the study show that multiple objective optimisation is a more complex approach minimizing the risk in the procedure of decision-making and maximizing the formulated objective.

Abbreviation

DM	Decision Making
FMODMP	Fuzzy Multiple Objective Decision-making Problem
FMOOP	Fuzzy Multiple Objective Optimisation Problem
FO	Fuzzy Optimisation
GMOOP	General Multiple Objective Optimisation Problem
MOOP	Multiple Objective Optimisation Problem

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